The worksheet explores the numerical solution of scalar ordinary differential equations. In particular we look at finite difference solutions of two point boundary value problems (BVP's).

Two Point BVP's

1. The differential equation for the displacement of a simply supported beam to due a uniform load is:

$$\frac{d^2y}{dx^2} = -\frac{1-x}{[1+(\alpha-1)x]^4}, \qquad y(0) = 0, \qquad y(1) = 0.$$

Solve the problem using the finite difference method (apply central difference formulas) using $\alpha = 1.5$ and $n = 21$. The exact solution is:

$$y_{ex} = -\frac{(3+2\alpha x-3x)x^2}{6(1+\alpha x-x)^2} + \frac{x}{3\alpha}.$$

Plot the finite difference and exact solutions on the same graph.

2. **One boundary value unknown**

If the boundary conditions are changed to

$$y(0) = 0, \qquad \left.\frac{dy}{dx}\right|_{x=1} = 0,$$

recompute and plot the finite difference solution. Repeat this procedure for the boundary conditions

$$\left.\frac{dy}{dx}\right|_{x=0} = 0, \qquad y(1) = 0.$$

3. Consider a metal rod of length 10m. Suppose that the temperature at one end of the rod (the lhs) is maintained at $T(0) = 0.5°C$ and that at the other end (the rhs) the temperature satisfies the condition:

$$\frac{dT}{dx} = 0.01, \qquad \text{at} \qquad x = 10.$$

If at each point x along the rod, there is a heat source of strength $T_s(x) = 2 + \cos x$, then the ODE describing the temperature (T) along the rod is:

$$\kappa\frac{d^2T}{dx^2} + \beta\frac{dT}{dx} + T = T_s(x),$$

Use $n = 101$, $\kappa = 1$ and $\beta = 1$ to compute the finite difference solution for the temperature, T. Plot the temperature, T, against x.