Systems of 1st order IVP's:

1. The differential equation describing the angular position of a mechanical arm is

$$\ddot{\theta} = \frac{a(b-\theta) - \theta\dot{\theta}}{1+\theta^2}$$

where $a = 100\,\mathrm{s}^{-2}$ and $b = 15$. If $\theta(0) = 2\pi$ and $\dot{\theta}(0) = 0$.

Solve the IVP using Euler's and Runge Kutta's method. Plot $\theta(t)$ and $\dot{\theta}(t)$ from $t = 0$ up to $t = 10$, using $\Delta t = 0.01$.

Letting

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix},$$

leads to the system of first order equations:

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} y_2 \\ \dfrac{a(b-y_1) - y_1\,y_2}{1+y_1^2} \end{bmatrix}.$$

Euler's Method

Create a script file **EulSys8.m** and the complementary function file **FuncT8.m** that apply Euler's method for systems to solve the IVP above.

Runge-Kutta method

Create a script file **RK_Sys8.m** that calls the function file **FuncT8.m** and applies the Runge-Kutta method for systems to solve the IVP above.

2. The problem of relaxation oscillations in lasers is described by the nonlinear system of differential equations:

$$\frac{dN}{dt} = P - \frac{N}{\tau_{decay}} - B\,n\,N,$$

$$\frac{dn}{dt} = -\frac{n}{\tau_{cavity}} + B\,n\,N,$$

Using

$P = 30$, $\tau_{decay} = 100$, $B = 3$, $\tau_{cavity} = 1/30$, $n(0) = 1$ and $N(0) = 1$,

use both Euler's method and the Runge-Kutta method to find and plot $N(t)$ and $n(t)$ up to $t = 3$ where $\Delta t = 0.01$.

Use the **subplot** command with 3 plots:

– In the first graph, plot N vs t

– In the second graph below the 1st, plot n vs t

– In the third graph below the 2nd, plot n (vertical axis) vs N (horizontal axis)